COMMUNICATIONS TO THE EDITOR

Heat-conduction Losses in Reactor Waste Basins

STANLEY H. JURY

University of Tennessee, Knoxville, Tennessee

In connection with one method of waste disposal investigated at Oak Ridge National Laboratories, it has been proposed that radioactive wastes be fed into large out-of-door basins. The basins would be formed in the surface of the earth, the excavation being about 30 ft. in depth and several hundred in breadth. The bottom and sides of the basin would be sealed with an appropriate asphalt base material which may or may not be provided with cooling ducts. A thermal insulating layer of earth or equivalent may or may not be poured over the asphalt seal, and then the rest of the basin would be filled with coarse rock or equivalent for deentrainment purposes.

During start-up of a basin the radioactivity would build up and gradually heat the basin and waste to the boiling point of the waste contained therein. Subsequently a steady state would be attained, when the water fed to the basin would equal the steam loss from the basin. Also, the rate of heat generation by decay would equal the rate of heat loss through steam generation and feed preheating plus the rate of heat conduction into the earth beneath the basin if it is assumed that there are no cooling ducts in the asphalt seal.

From an engineering viewpoint it is easy to estimate for design purposes the various heat-flow rates if the loss into the earth is known. This loss is difficult to determine and is really the subject of this paper for the case wherein cooling ducts are not involved.

ANALYSIS OF STEADY STATE HEAT CONDUCTION IN EARTH

The actual plan of the basin was to be rectangular in shape with sloping sides which changed in pitch as they approached the bottom of the basin. Such a geometry complicates boundary conditions, and therefore it was considered necessary for mathematical convenience only, to adopt an idealized geometry which closely approximated the original. The basin is so shallow, with its length and breadth being approximately the same, that for purposes of heat conduction into the earth the heat losses should be essentially the same as for basin of zero depth and an area in contact with the earth which is circular and equal to the area in contact with the earth for the original pool, i.e., approximately 189,500 sq. ft. Thus the circular model basin has a radius R equal to 245.5 ft.

The mathematical problem involves a disk of radius R maintained at a temperature $\tau_b = 230°F.$, the disk being laid flat on the surface of the earth, which is assumed to be flat and of infinite extent behind this surface. If r is an arbitrary distance from the center of this basin to any point in the surface and Z is the normal distance from the surface of the earth of a point in the earth, then the differential conduction equation in cylindrical coordinates is

$$\frac{\partial^2 \tau}{\partial r^2} + \frac{1}{r}\frac{\partial \tau}{\partial r} + \frac{\partial^2 \tau}{\partial Z^2} = 0 \qquad (1)$$

The limiting conditions are

$$\tau(r, 0) = \tau_b \qquad 0 \leq r \leq R \quad (2)$$

$$k\left(\frac{\partial \tau}{\partial Z}\right)_{Z=0} - h(\tau - \tau_1) = 0 \quad r > R \quad (3)$$

where τ_1 is the temperature of the air over the earth's surface and h is the radiation-convection film coefficient for the cooling winds which naturally circulate over the surface of the earth; k is the thermal conductivity of the earth.

If one defines

$$T = \frac{\tau - \tau_1}{\tau_b - \tau_1} \qquad (4)$$

then Equation (1) becomes

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial Z^2} = 0 \qquad (5)$$

and Equation (2) may be written

$$T(r, 0) = 1 \quad 0 \leq r \leq R \qquad (6)$$

While for Equation (3) one has

$$k\left(\frac{\partial T}{\partial Z}\right)_{Z=0} - hT = 0 \quad r > R \quad (7)$$

This problem was run off on the Oracle, a digital computer, to avoid insuperable mathematical difficulty. To do so, the symmetry about $r = 0$ was recognized and a plane was envisioned as connecting the $r = 0$ axis with an arbitrarily chosen radial line in the surface of the earth which was $5R$ in length. The axis was also drawn to extend $5R$ from $Z = 0$. This axis and the radial line were each divided into twenty-five equal segments, each segment being $R/5$ long. The points of division were used then to establish a symmetrical 25- × 25-point grid.

The differential problem was translated into the equivalent finite-difference problem with the finite increment chosen as equal to $R/5$. The Oracle thus computed the value of T for each of the 625 points of the grid. The result, of course, is approximate because, among other things, it is assumed that all earth beyond $5R$ along r or Z does not materially alter the heat conduction in the earth at the basin; i.e., the temperature distribution beyond $5R$ is essentially uniform. The results substantially satisfied this criterion.

Two cases of the foregoing problem were run off on the Oracle. The first case involved $h/k = 92.2$ reciprocal ft. The second case involved $h/k = 9.22$ reciprocal ft.

RESULTS

The heat conduction from the basin into the earth can be estimated with the formula

$$\begin{aligned}\frac{q}{\tau_b - \tau_1} &= k\pi\left(\frac{R}{2\cdot 5}\right)^2\left[T(r=0, Z=0) - T\left(r=0, Z=\frac{R}{5}\right)\right]\frac{5}{R} \\ &+ k\pi\left[\left(\frac{3R}{2\cdot 5}\right)^2 - \left(\frac{R}{2\cdot 5}\right)^2\right]\left[T\left(r=\frac{R}{5}, Z=0\right) - T\left(r=\frac{R}{5}, Z=\frac{R}{5}\right)\right]\frac{5}{R} \\ &+ k\pi\left[\left(\frac{5R}{2\cdot 5}\right)^2 - \left(\frac{3R}{2\cdot 5}\right)^2\right]\left[T\left(r=\frac{2R}{5}, Z=0\right) - T\left(r=\frac{2R}{5}, Z=\frac{R}{5}\right)\right]\frac{5}{R} \\ &+ \cdots\cdots\cdots \\ &+ k\pi\left[\left(\frac{10R}{2\cdot 5}\right)^2 - \left(\frac{9R}{2\cdot 5}\right)^2\right]\left[T(r=R, Z=0) - T\left(r=R, Z=\frac{R}{5}\right)\right]\frac{5}{R}\end{aligned}$$

where

q = B.t.u./hr. heat loss from basin into earth

τ_b = 230°F.

τ_1 = 60°F. average year-around out-of-door temperature

For case 1

q = 8,000 B.t.u./hr.

(Continued on page 9M)

(*Continued from page 143*)

For case 2

$q = 78{,}000$ B.t.u./hr.

Another quantity of interest is the thermal gradient in the earth at the center of the basin and $Z = 0$. This quantity may be estimated from the formula

$$\text{Gradient} = [T(r = 0, Z = 0) - T(r = 0, Z = R/5)][r_b - r_1]5/R$$

For case 1

Gradient = 0.485°F./ft.

For case 2

Gradient = 0.473°F./ft.

The gradient values are significant in that if one had, for example, an asphalt that would not stand more than 220°F., one would need 21.2 ft. of earth in case 2 to insulate the boiling solution from the asphalt seal, and this figure is obviously prohibitive. The answer is not significantly different for case 1. In this regard, it should be stated that case 2 is the more realistic of the two because of the k value involved.

The values of T for the 625 grid points were considered too voluminous to be included in this paper.

CONCLUSIONS

The conclusions reached were that the steady state heat losses from the bottom of the basin would be negligible compared with the rate of radio heat generation. However, the temperature gradient at the bottom center of the pool is such that the pool would have practically to be filled with insulating material in order to ensure a low enough temperature in the bottom seal so as not to soften it and cause leakage.

The basin project was discontinued before data could be obtained. No comparable work seems to have been reported to date.

BOOKS

Unit Operations of Chemical Engineering. Warren L. McCabe and Julian C. Smith. McGraw-Hill Book Company, Inc., New York (1956). 945 pages. $10.50.

This book covers the same topics as the well-known earlier work "Elements of Chemical Engineering" by W. L. Badger and W. L. McCabe, the most recent edition of which was published in 1936. Newer unit operations, such as adsorption, ion exchange, and dialysis, are not included. However, the text of all sections has been completely rewritten to incorporate the added understanding of unit operations developed in the past twenty years. Old subject matter has been eliminated or rearranged to be consistent with new material.

The level of treatment is designed for undergraduate students, but the discussion has been carried to the point where students will find an easy transition to more advanced chemical engineering texts. In addition, new approaches have been incorporated; for example, the boundary-layer concept is introduced early in the fluid mechanics study, the theory of diffusion is based on the relative-velocity method, and the discussion of the separate mass transfer operations is preceded by a study of principles common to all.

In order to provide material for a three-semester sequence for unit operations and include the developments of recent years, while at the same time maintaining an adequate treatment of theory and equipment, the authors increased the length of the book from 660 to 945 pages. This increase was due primarily to the substantial expansion of the sections on fluid mechanics and flow of heat and the general approach to mass transfer, but was also due to the enlarged coverage of the distillation, mixing, and extraction sections.

To students, teachers, and practicing engineers alike, this book should be a welcome text, especially for those accustomed to the 1936 book. Although some of the solved problems and diagrams from the previous text have been included in modified form, most are replaced by new illustrations. All the unsolved problems are new. Mass transfer coefficients have been standardized to the Drew-Colburn coefficient. As before, a discussion of pertinent equipment aug-